IB Physics IA May 2025

Topic: How does the drop height of a ball affect the time it remains submerged under water?

Word Count: 2895

Introduction

When I was young, my friends and I would often play around in the swimming pool in the summers. One of the many activities we did in the pool was called a “cannonball”—essentially curling oneself up in the shape of a ball and jumping into the pool. It was on one such summer day that I jumped from a 10-metre diving board and plunged deep into the pool. I vividly remember almost touching the pool floor and staying at that ‘positional minima’ for a brief 5-6 seconds. Indeed, this piqued my interest I found myself pondering the larger implications of said topic. On further investigation, I found this topic to be a part of a famous sub-discipline in physics known as fluid dynamics.

Fluid dynamics is observed in several aspects of the world: air conditioners, smoke pumping out of the exhaust of your car, as well as designing efficient water management infrastructure. After reading up on an academic paper on motion dynamics of an object in a fluid, I found that the motion of an object in water is indeed affected by its drop height[1]. Thus, I conducted a pilot study on the research question: **How does the drop height of a ball affect the time it remains submerged underwater?** This study had two goals: to get accustomed to the tracker software and foresee any problems which could arrive in the final experiment; for the pilot, I used a simple release mechanism at two heights. This choice of apparatus was optimal since it gave me an idea of whether my dependent variable (time submerged) had noticeable variations for smaller changes in heights (0.3m). Additionally, while using the tracker software for the pilot, I noticed that due to a fold in the rim of my water bucket, it was nearly impossible to track the position of the ball for a few frames when it was passing through the rim of the bucket. It was also observed that dropping the ball at different distances from the centre of the bucket yielded different results for the time that the ball would remain submerged underwater at a constant drop height. Finally, changes to the apparatus were a necessity because the current apparatus limited the extent to which the independent variable (drop height) could be varied. Thus, the aim of this investigation is to develop a theoretical relationship to be able to determine the effect of the variation of the independent variable on the dependent variable, and collect data to support or disprove the same; ultimately answering the research question: **How does the drop height of a ball affect the time it remains submerged underwater?**

[1] Xiang, Gong, et al. “Motion Dynamics of Dropped Cylindrical Objects in Flows after Water Entry.” *Ocean Engineering*, vol. 173, Feb. 2019, pp. 659–71, https://doi.org/10.1016/j.oceaneng.2019.01.010. Accessed 12 May 2021.

Background Information

When a body is dropped in a fluid, say water, it experiences three main forces: a downward force due to gravity, an upward force due to buoyancy, and a force opposing the motion of the body in the fluid called drag force. Throughout the body's motion in the fluid, the force due to gravity remains constant; however, the drag force and buoyant force experienced by the body increases initially, and at a certain moment the 3 forces balance out leading the ball to reach a terminal velocity in the fluid. Since the velocity is now constant, the drag force should remain unchanged. Shouldn't this mean that every object will sink to the bottom of the fluid?

Intuitively, yes. But, in reality, the body (let's take the tennis ball's example) experiences something known as the "pop-up" effect. This effect dictates that the motion a ball when dropped into a fluid (water in this case) is not only influenced by the three forces stated above but also complex fluid dynamics phenomenon like vortex shedding and creation of wakes. For our case, the formation of said wakes and vortices creates an upward pressure zone and assists in lowering the resistance that the ball faces while rising[2]. This is why the ball does not sink.

Buoyant Force

Buoyancy is the tendency of an object to float in a fluid. Thus, the upward force exerted on an object wholly or partially immersed in a fluid is known as the buoyant force (also known as upthrust). [3]

It is given by the formula:

$$\boldsymbol{F_b} = \boldsymbol{V} \times \rho \times \boldsymbol{g}$$ [4]

Where,

$F_b = Buoyant\ force$

$V = Volume\ of\ the\ displaced\ fluid$

$\rho = \text{Density of the fluid (For water it is 997 kg m}^{-3}\text{)}$

$g = acceleration\ due\ to\ gravity\ (9.81\ m\ s^{-2})$

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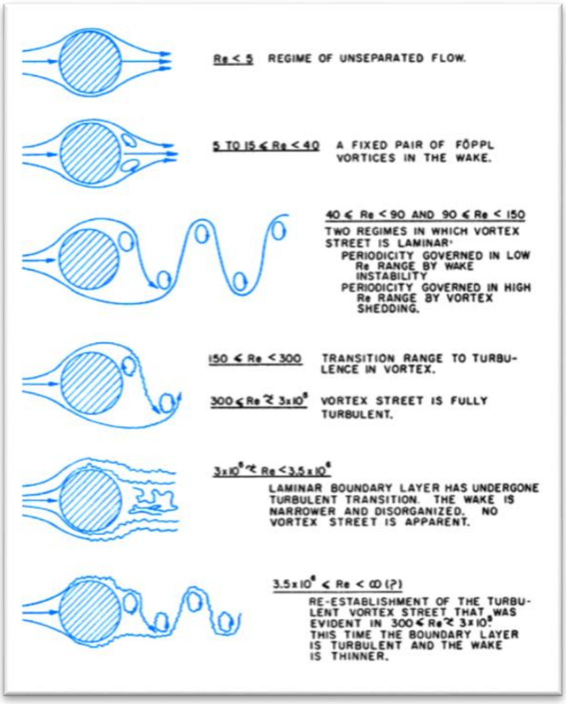
[2] University, Utah State. "The Pop-up Effect: Why Buoyant Spheres Don't Always Leap out of the Water." *Utah State Today*, 3 Nov. 2016, www.usu.edu/today/story/the-pop-up-effect-why-buoyant-spheres-dont-always-leap-out-of-the-water.

[3] Byju's. "Buoyant Force." *BYJUS*, Byju's, 10 June 2019, byjus.com/physics/buoyant-force/.

[4] "Deriving Buoyant Force from First Principles." *Physics Stack Exchange*, physics.stackexchange.com/questions/540241/deriving-buoyant-force-from-first-principles.

Wakes and Vortex Structures

Wakes and vortex structures are fluid dynamics phenomena that occur when an object, like a tennis ball, moves through a fluid such as water. As the ball moves through the fluid, it disturbs the surrounding water, creating a turbulent region behind it called the wake. This wake is characterized by swirling flows and vortex structures, which are rotating regions of fluid. Vortex shedding occurs when alternating vortices form and detach from opposite sides of the ball. These disturbances increase drag and create pressure differences around the ball, significantly influencing its motion and flow behaviour through the water.



[5]

Fig 1. Overview of different vortex shedding regimes

Reynolds Number

Reynolds number (Re) is a dimensionless quantity used in fluid mechanics to characterize the flow of a fluid. It is the ratio of inertial forces to viscous forces and is given by the formula[6]:

$$Re = \frac{(\rho u L)}{\mu}$$

[5] "What Is Microfluidic Vortex Shedding (MVS)." *Indee Labs*, www.indeelabs.com/articles/what-is-microfluidic-vortex-shedding.

[6] Engineering ToolBox. "Reynolds Number." *Engineeringtoolbox.com*, 2019, www.engineeringtoolbox.com/reynolds-number-d_237.html.

Where:

ρ is the density of the fluid,

u is the characteristic velocity of the fluid ($here, u = velocity\ of\ the\ ball\ (v)$)

L is a characteristic length (here, $L = diameter\ of\ the\ ball\ (d)$)

μ is the dynamic viscosity of the fluid.

Thus, for our experiment the formula for Re is given as:

$$Re = \frac{\rho \times v \times d}{\mu}$$

Estimating the Reynolds number at room temperature for a ball being dropped into water from a height of 1m (i.e. impact velocity of 4.43 m s^{-1}) we get,

$$Re = \frac{1000 \times 4.43 \times 0.065}{0.001} \approx 2.8 \times 10^5$$

Drag Force

The drag force is a type of force that opposes the motion of a body in a fluid.

We found that the Reynolds Number for a ball being dropped into water at room temperature from heights greater than 1m to be high (i.e. $Re \geq 2.8 \times 10^5$); thus, we can deduce that there is turbulent flow. Since Stokes' Law assumes laminar flow, we cannot use it to calculate drag force in this scenario. Hence, the appropriate equation for drag force is given by:

$$F_d = 0.5 \times \rho \times v^2 \times C_d \times A$$ [7]

Where,

$F_d = drag\ force$

$\rho = density\ of\ fluid$

$v = speed\ of\ the\ object\ relative\ to\ the\ fluid$

$C_d = drag\ coefficient$

$A = cross\ sectional\ area\ of\ the\ ball\ perpendicular\ to\ the\ direction\ of\ motion$

However, in the presence of a turbulent wake, the drag coefficient is reduced to C'_d. Where,

$C'_d = C_d(1 - k_w)$

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[7] Lumen Learning. "Drag Forces | Physics." *Courses.lumenlearning.com*, 2022, courses.lumenlearning.com/suny-physics/chapter/5-2-drag-forces/.

Here, k_w is wake factor representing the reduction in drag coefficient due to a turbulent wake. This happens because the wake modifies the flow around the ball, which creates an area of lower pressure and reduces resistance.

Vortex Shedding

As the ball moves downward in the water, it disrupts the flow and creates alternating vortices on either side of the ball. These vortices are shed in a periodic manner and create a pattern of swirling fluid behind the ball, often referred to as a Von Kármán vortex street.

Vortex shedding can induce an upward lift force F_L given by[8]:

$$F_L = \rho_w v \Gamma^{9}$$

Where, ρ_w is the fluid density, v is the velocity of the ball, and Γ is called circulation which depends on the vortex structure and can be written as a function of the velocity and diameter of the ball.

Theoretical model

Using newton's 2nd Law of motion,

$$\sum F = ma^{10}$$

For a ball moving in water,

$$\sum F = F_b - F_g - F'_d + F_L$$

$$\therefore m\frac{dv}{dt} = \rho V g - mg - \frac{1}{2}C'_d \rho A v^2 + \rho v \Gamma^{11}$$

Solving this differential equation is outside the scope of this study since it will require complex mathematical modelling. Solving this equation, however, would help model time as a function of impact velocity of the ball when it reaches the surface of the water where the impact velocity is nothing but $\sqrt{2gh}$.

[8] Rigo, François, et al. "Generalized Lift Force Model under Vortex Shedding." *Journal of Fluids and Structures*, vol. 115, 1 Nov. 2022, pp. 103758–103758, https://doi.org/10.1016/j.jfluidstructs.2022.103758. Accessed 6 Feb. 2024.
[9] "Ideal Lift of a Spinning Ball | Glenn Research Center | NASA." *Glenn Research Center | NASA*, 18 July 2024, www1.grc.nasa.gov/beginners-guide-to-aeronautics/ideal-lift-of-a-spinning-ball/#:~:text=The%20equation%20given%20above%20describes. Accessed 13 Sept. 2024.
[10] Hall, Nancy. "Newton's Laws of Motion." *Glenn Research Center*, NASA, 27 June 2024, www1.grc.nasa.gov/beginners-guide-to-aeronautics/newtons-laws-of-motion/.
[11] More on this in the appendix

Research Question & Hypothesis

Research Question: How does the drop height of a ball affect the time it remains submerged underwater?

The hypothesis for this study is that the time submerged will increase as the height of the ball from the water surface increase. This is because a higher height will yield a higher impact velocity which will take longer to come to rest. Additionally, after a certain height the impact velocity will become equal to the terminal velocity of the ball in air; thus, the time submerged too should reach a maximum limit at that height.

This is supported by the theoretical model, we can deduce that $t_{submerged}$ for small values h will increase approximately as $h^{\frac{1}{n}}$: where n can be determined via experimentation ; whereas, for large values of h, $t_{submerged}$ might increase at a negligible rate, almost—but, not quite—approaching a plateau.

Independent and Dependent Variables

Table 1. Independent and Dependent Variables

Variable Name	Variable Type	Units
Drop height of a ball above the surface of the water (h)	Independent	Metre (m)
Time submerged underwater (t)	Dependent	Seconds (s)

Control Variables

Table 2. Control Variables

Control Variable	Reason for Control	Method of control
Ball Used	The ball's mass, size, and shape directly affect the forces acting on it, such as buoyancy and drag. Any change in these properties would alter the submersion time, leading to inconsistent results.	Use the exact same tennis ball in every trial. Ensuring the surface properties of the ball are consistent between trials.[12]
Temperature of the experimental setup	Temperature can affect the density of water and the dynamic viscosity of the fluid,	Conduct the experiment in an air-conditioned room in order to maintain a room temperature of

[12] Chadwick, S. G., and S. J. Haake. "The drag coefficient of tennis balls." The engineering of sport: research, development and innovation (2000): 169-176.

	both of which influence the drag force on the ball since they can vary the Reynolds Number.	20°C and monitor the water and room temperature with a thermometer between trials.
Initial Velocity	Irregular initial velocity would significantly impact the submersion time—compromising the experiment's integrity.	The ball should be tied to a thread and hanged from a rigid support. Similar to a pendulum at its equilibrium position, the forces of tension and weight of the ball would balance out. Hence, when the string is finally cut the only force acting on the ball would be its weight due to gravity—giving it an initial velocity of 0 m s^{-1}.
External Force on the ball	Same reason as initial velocity.	Same method as initial velocity.
Camera used to record the experiment	Different cameras or settings can result in different frame rates, potentially affecting how accurately the timing of the submersion is recorded.	The same 60fps 24megapixel camera is used throughout the experiment.
Volume of Water	The amount of water affects the depth and the buoyant force acting on the ball.	The bucket used has measurements at different levels demarcating the volume of water at that level. The same water level is maintained throughout the experiment i.e. 15 litres of water.
Spin in the Drop of the Ball	It may increase drag due to angular velocity or change how the ball interacts with the water, leading to inconsistent results.	Ensuring that the ball is completely at rest before cutting the thread.

Research Design

Fig 2. Experimental setup

Procedure

A tennis ball is tied to an inextensible string of negligible mass and hanged from a fixed rigid support—like a metal rod. Afterwards, a bucket filled with 15litres of water is placed directly underneath the ball. It is necessary to ascertain that at rest the ball has no spin, and it is directly above the centre of the water bucket. A camera is then placed at the same level as the water—to avoid parallax error. Finally, the string is cut and the motion of the ball is recorded through the camera. The drop height of the ball can be varied by changing the length of the string. Repeat this process 5 times at 5 or more different heights. (Note: the room is air conditioned in order to control the room temperature.)

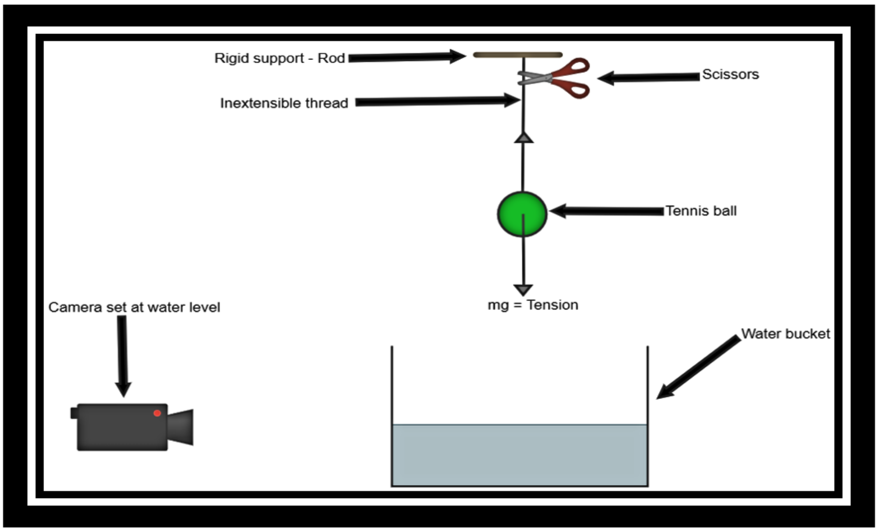

Fig 3. Schematic Representation of the setup

[13]

The footage is uploaded on the tracker software. Wherein, the bucket is set as the reference frame and the motion of the ball is tracked frame by frame for each clipping. It is important to set the centre of the bucket at the level of the water as the (0,0) or origin point in the tracker software. A graph of vertical position vs the time is then sketched on the tracker software. To find the actual time that the ball remains submerged, one must locate the first two x-intercepts of the graph—the time at which the ball first goes below the surface of the water and the time when it resurfaces—and subtract t_1 from t_2. Using this method, we can collect the data required for the graphical analysis which will be addressed in a later section.

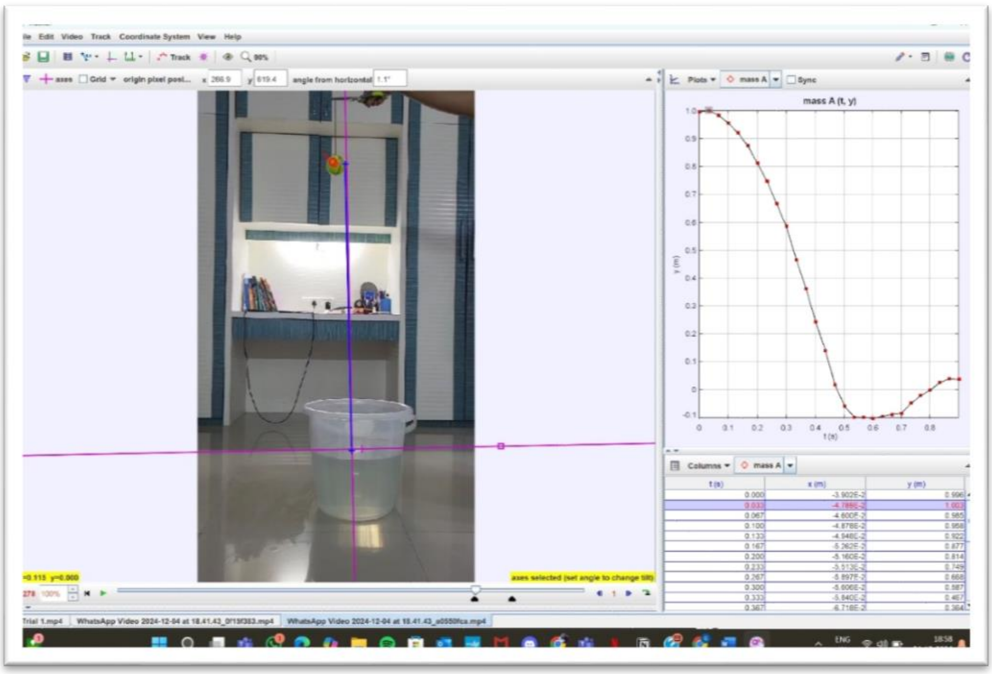

Fig 4. Screenshot from video analysis software

[13] www.chemix.org

Raw Data

Table 3. Raw Data measuring release height of ball (metres) & time submerged underwater (seconds)

Release height (m) (± 0.01m)	**Time Underwater (s) (± 0.001s)**				
	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**
0.20	0.259	0.272	0.270	0.284	0.263
0.40	0.292	0.310	0.298	0.283	0.307
0.60	0.341	0.331	0.326	0.326	0.327
0.80	0.349	0.347	0.352	0.348	0.367
1.00	0.385	0.370	0.379	0.391	0.381
1.20	0.402	0.399	0.399	0.400	0.398
1.40	0.433	0.422	0.433	0.430	0.443
1.60	0.446	0.454	0.450	0.466	0.453

Processed Data & Error Analysis

Table 4. Processed data with error calculations

Release height (m) (± 0.01m)	**Time Underwater (s) (± 0.001s)**						
	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**	**Mean time underwater (s) (± 0.001s)**	**Uncertainty in time underwater (s) (± 0.001s)**
0.20	0.259	0.272	0.270	0.284	0.263	0.270	0.013
0.40	0.292	0.310	0.298	0.283	0.307	0.298	0.014
0.60	0.341	0.331	0.326	0.326	0.327	0.330	0.008
0.80	0.349	0.347	0.352	0.348	0.367	0.353	0.010
1.00	0.385	0.370	0.379	0.391	0.381	0.381	0.011
1.20	0.402	0.399	0.399	0.400	0.398	0.400	0.002
1.40	0.433	0.422	0.433	0.430	0.443	0.432	0.011
1.60	0.446	0.454	0.450	0.466	0.453	0.454	0.010

The error in the time submerged underwater at a certain height is calculated by the following formula:

$$\Delta t = \frac{t_{max} - t_{min}}{2}$$

For example, at height 1.6m we get $t_{max} = 0.466s$ and $t_{min} = 0.446s$. Therefore,

$$\Delta t = \frac{0.466 - 0.446}{2} = \pm 0.010s$$

Since the least count of time is 0.001s and the uncertainty does not exceed 3 decimal places, there is no need for rounding off in this calculation.

The mean time submerged underwater, on the other hand, was calculated as follows:

$$Mean\ Time\ Submerged\ underwater = \frac{\sum_{i=0}^{5} Trial\ i}{5}$$

Thus, for the same height (1.6m) we get,

$$Mean\ Time\ Submerged\ underwater = \frac{0.446 + 0.454 + 0.450 + 0.466 + 0.453 + 0.454}{5} = 0.454s$$

Graphical Analysis

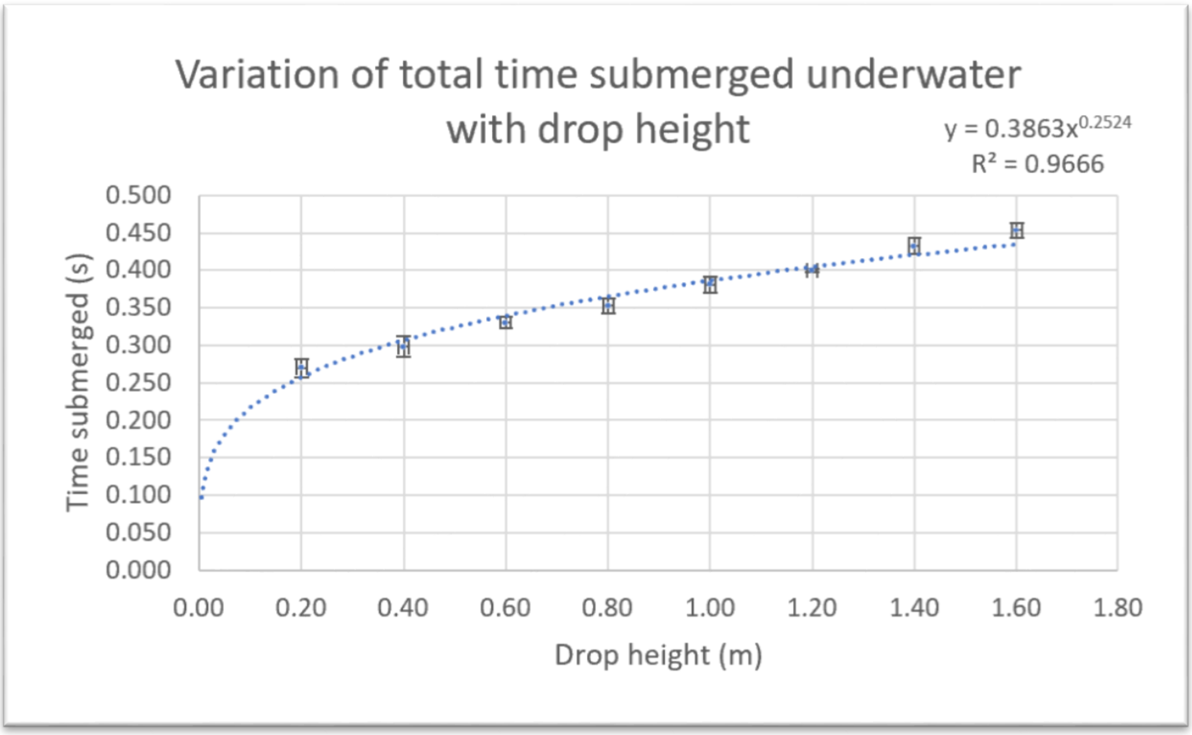

Fig 5. Graphical Representation of experimental data

The graph above shows the variation of time submerged underwater for a tennis ball with its drop height above the surface. The data points collected from experimentation are plotted on an Excel scatterplot, specifically the mean values of the dependent variable against the independent variable. Then the error margins calculated for the dependent variable as well as the fixed error margins for the independent variable are plotted in the form of horizontal and vertical error bars. It is important to note that the error bars plotted are incredibly small this is because we used a computer software that analyses the ball frame-by-frame to determine the time submerged under water.

A trendline is also plotted, and, as can be seen in the graph, the line passes through all points within their error margins. This claim is further reinforced by the high R^2 value of 0.9666. From the hypothesis (based on the derivation in the appendix,) we get the relation $t \propto h^{\frac{1}{n}}$ where t is the time submerged underwater, h is the drop height above the water surface, and the radical $\frac{1}{n}$ tells us that the power function shows growth. From the hypothesis we can see that the trendline assumes the relationship described in the hypothesis where the constant of proportionality is 0.3 and the value of n is approximately 4.

Where the hypothesis falls short, however, is in the claim that if the ball is dropped from a height wherein it reaches terminal velocity in air before impact with the water surface, dropping the ball from an even greater height will result in an equivalent time submerged underwater. This is clearly not true since the trendline is of a growth function. The reason the ball stays submerged longer when dropped from a greater height, even after reaching terminal velocity in air, lies in its accumulated momentum and kinetic energy. While the velocity at water entry remains constant due to terminal velocity, the ball continues to fall for a longer duration, increasing its total momentum and kinetic energy. Upon entering the water, the drag and buoyant forces must counteract this increased energy, causing the ball to decelerate more gradually. This results in a longer submersion time as the ball takes more time to lose its energy in the water. This explanation can also be verified by the nature of the graph: it doesn't have a horizontal asymptote but after a certain value of height (when the ball reaches terminal velocity in air) the graph increases at an increasingly small rate.

Another peculiar observation in the graph is the prominent y-intercept. Intuitively, it must indicate a systematic error, but the reason for the ball being submerged underwater for a short period of time even when placed directly into the water is because of the bobble effect. That is, when a ball is placed on water, gravity initially pulls it downward, causing it to sink slightly. As this happens, buoyancy increases until it balances gravity, stabilizing the ball. If the ball has momentum, it temporarily exaggerates the submersion before equilibrium is restored. Surface tension initially resists the ball's entry but later aids in stabilizing it on the surface. The surrounding water reacts to the disturbance, creating damping oscillations as the forces balance and the ball reaches a state of equilibrium. Because of this, a small y-intercept is to be expected.

Evaluation

Despite its strengths, the experiment has some limitations related to the apparatus and the range of drop heights. The use of a small bucket introduces edge effects, which may influence the flow of water and affect the natural trajectory of the ball, especially at higher drop heights. This limitation restricts the fluid dynamics to an artificial setup, potentially skewing results. Additionally, the range of heights tested, though adequate for observing trends, does not extend far enough to conclusively evaluate the impact of terminal velocity on submersion time.

The study faces inherent limitations due to simplified assumptions in its theoretical model. The omission of certain forces, such as lift caused by vortex shedding and atmospheric drag, limits the accuracy of the predictions for the ball's motion at higher velocities.

To address the weaknesses and limitations, several improvements could be made. Replacing the bucket with a larger water container or conducting the experiment in a swimming pool would minimize edge effects and allow for more natural fluid behaviour. Expanding the range of drop heights beyond 1.6 meters would provide better insights into how terminal velocity influences submersion time. Incorporating higher-resolution cameras or advanced sensors for data collection could further enhance measurement accuracy. Increasing the number of trials at each height would reduce statistical uncertainties, ensuring more reliable results.

Furthermore, although the apparatus chosen was appropriate for the given experiment, uncertainty margins could further be improved by eliminating human involvement. For instance, the string had to be cut by a human which might've added a jerk or additional tension to it. Instead, a different launch apparatus could have been setup using an electromagnetic clamp which could help monitor the time and height more accurately, Unfortunately, the costs of this setup would have been significantly more expensive, and considering the budget limitations of INR 1,000 for this investigation it was not an economically feasible option.

The investigation can be extended by experimenting with balls of varying sizes, shapes, and surface to determine how physical properties impact fluid resistance and motion. Additionally, introducing spin to the ball would help analyse the role of angular momentum in influencing drag and lift forces. Furthermore, rectifying the theoretical model to incorporate complex fluid dynamics phenomena, such as vortex shedding and atmospheric drag, could provide a more intricate relationship compared to the generalized one offered in this study.

Conclusion

In conclusion, this investigation has successfully explored the relationship between the drop height of a ball and the time it remains submerged underwater. The results confirm that higher drop heights lead to increased submersion times due to greater impact velocities and kinetic energy, which require more time for the forces of drag and buoyancy to counteract. The findings align with theoretical predictions, showing a non-linear relationship that grows at a diminishing rate as height increases, without plateauing. Despite the limitations to the study due to edge effects and lack of trials at high drop heights, the study provides a deep insight into the research question by providing a theoretical derivation as well as a well thought out experiment. The research could further me extended by incorporating larger setups to minimize boundary effects, testing a broader range of heights, and exploring the influence of spin or different object properties.

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Appendix

Derivation of time submerged as a function of drop height—neglecting lift force due to vortex shedding:

1

Derivation – Time submerged

Pt I:

Calculate Impact Velocity

$u = 0$

$a = g$

$h =$ Independent Variable

$$V_{impact}^2 = u^2 + 2gh$$

$$V_{impact} = \sqrt{2gh} \quad \ldots [i]$$

Pt II:

Calculate time submerged underwater

$$t_{total} = t_{down} + t_{up} \quad \ldots [ii]$$

Calculate t_{down}

$$\Sigma F = ma = F_g - F_b - F_d = ma$$

$$a = \frac{dv}{dt}$$

$$\therefore mg - \rho_w V_b g - \frac{1}{2} C_d \rho_w A v^2 = m\frac{dv}{dt}$$

$$m\frac{dv}{dt} = g(m - \rho_w V_b) - \frac{1}{2} C_d \rho_w A v^2$$

Date

(2)

Let $g(m - \rho_w V_b) = F_{eff}$ ...[iii]

Let $\frac{1}{2} C_d \rho_w A = k$ ...[iv]

$$\therefore m\frac{dv}{dt} = F_{eff} - kv^2 \Rightarrow \frac{dv}{dt} = \frac{F_{eff}}{m} - \frac{k}{m}v^2$$

$$\frac{dv}{\frac{F_{eff}}{m} - \frac{k}{m}v^2} = dt$$

[v]... Let, $\frac{F_{eff}}{m} = \alpha$

[vi]... Let $\frac{k}{m} = \beta$

$$\int \frac{dv}{\alpha - \beta v^2} = \int dt$$

$$t = \frac{1}{\sqrt{\alpha\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot v\right) + C$$

At $t = 0$, $v = V_{impact}$ (V_{imp})

$$0 = \frac{1}{\sqrt{\alpha\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot V_{imp}\right) + C$$

$$C = -\frac{1}{\sqrt{\alpha\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot V_{imp}\right)$$

$$t = \frac{1}{\sqrt{\alpha\beta}}\left[\tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot v\right) - \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot V_{imp}\right)\right]$$

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(3)

The ball comes to rest at $V=0$ and $t=t_{down}$

$$t_{down} = \left| \frac{1}{\sqrt{\alpha\beta}} \left[0 - \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot v_{imp}\right)\right] \right|$$

$$t_{down} = \frac{1}{\sqrt{\alpha\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha}} \cdot v'_{imp}\right)$$

$$\therefore t_{down} = \frac{m}{\sqrt{F_{eff}\, k}} \tanh^{-1}\left(\sqrt{\frac{k}{F_{eff}}} \cdot V_{imp}\right) \ldots [vii]$$

Calculate t_{up}

$$m\frac{dv}{dt} = F_b - F_g - F_d = \rho_\omega V_b g - mg - \frac{1}{2} C_d \rho_\omega A v^2$$

Let $F_{eff,up} = g(\rho_\omega V_b - m)$

Let $k = \frac{1}{2} C_d \rho_\omega A$

$$m\frac{dv}{dt} = F_{eff,up} - kv^2$$

$$\frac{dv}{\frac{F_{eff,up}}{m} - \frac{k}{m}v^2} = dt$$

Let $\alpha_{up} = \frac{F_{eff,up}}{m}$

Let $\beta = \frac{k}{m}$

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$$\int \frac{dv}{\alpha_{up} - \beta v^2} = \int dt$$

$$t = \frac{1}{\sqrt{\alpha_{up}\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha_{up}}} \cdot v\right) + C$$

$V = 0$ at $t = 0$ $\therefore C = 0$

At the surface, the forces balance out. And, upon reaching the surface the ball will come to rest (assumption)

Hence, as $\lim_{t \to t_{up}}$, $\lim_{v \to 0}$

Thus,

$$t_{up} = \frac{1}{\sqrt{\alpha_{up}\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{\alpha_{up}}} \cdot \lim_{v \to 0}\right)$$

$\therefore t_{up} \approx 0$

$\therefore t_{total} = t_{down} + t_{up} \approx t_{down}$

$$\therefore t_{total} = \frac{m}{\sqrt{F_{eff}\, k}} \tanh^{-1}\left(\sqrt{\frac{k}{F_{eff}}} \cdot v_{imp}\right)$$

$\sqrt{2gh}$

~~total~~

~~$\therefore t_{total} = \frac{m}{\;} \;\; t_{total}$~~

$\therefore t_{total} \propto h^{n}$

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Likely graph: